Generalized Coordinates Part II

(B.Sc. (Physics) Part-I, Paper-I, Group-B)

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"If a man never contradicts himself, the reason must be that he virtually never says anything at all."

— Erwin Schrödinger (1887-1961)

In the last lecture note, we have discussed generalized coordinates and types of constraints on the motion of a particle. This class note is dedicated to elaborate more about generalized coordinates.

If there is any constraint on the motion of a particle, then Newton's equations of motion in the terms of Cartesian coordinates do not incorporate the constraint. To include these constraint one, therefore, has to add additional conditions and solve them simultaneously with the dynamical equations. To overcome this additional difficulty on solving dynamical equations people have introduced the concept of generalized coordinates which are customarily represented by a set of independent coordinates

$$q_1, q_2, q_3, ...q_n,$$

or q_k where $k = 1, 2, 3, ..., n$ with n is the number of degrees of freedom of the system. Below we list two basic points on generalized coordinates for a given system S of N particles: (i) Generalized coordinates are independent variables, i.e., there is no functional relations connecting them. (ii) For the given system S, the position vectors $\{\mathbf{r}_i\}$ of the particles must be known functions of independent variables $\{q_k\}$,

$$\mathbf{r}_i = \mathbf{r}_i(q_1, q_2, ..., q_n) \qquad (i = 1, 2, ..., N) \tag{1}$$

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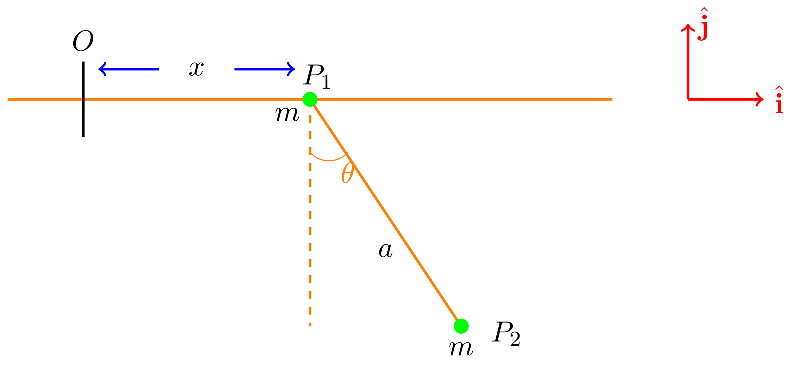

FIG. 1: x and θ are generalized coordinates for the system.

The concept of generalized coordinates seems to be abstract but is easy to implement. Usually, these coordinates are chosen to be displacements or angles which appear normally in the problem.

Question. Let S be the system shown in Figure (1) which consists of two particles P_1 and P_2 connected by a light rigid rod of length a. The particle P_1 is constrained to move along a fixed horizontal rail and the system moves in the vertical plane through the rail. Select generalized coordinates for this system and obtain expressions for the position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ in terms of these coordinates.

Solution We consider here the variables x, θ which are independent, i.e., they are not connected by functional relation. Therefore, $\{x, \theta\}$ are the generalized coordinates of the system. Now the position vectors for particles P_1 and P_2 in terms of generalized coordinates are, respectively, given by

$$\mathbf{r}_1 = x\hat{\mathbf{i}}, \tag{2}$$

$$\mathbf{r}_1 = (x + a\sin\theta)\hat{\mathbf{i}} - a\cos\theta\hat{\mathbf{i}}, \tag{3}$$

which are the Eqs. (1) for the given system in terms of the choice of generalized coordinates x and θ.

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[1] H. Goldstein, C. Poole, and J. Safko, *Classical Mechanics*, Addison-Wesley, (2000).